

Exercise 96

Suppose $y = f(x)$ is a curve that always lies above the x -axis and never has a horizontal tangent, where f is differentiable everywhere. For what value of y is the rate of change of y^5 with respect to x eighty times the rate of change of y with respect to x ?

Solution

$$\frac{d}{dx}(y^5) = 80 \cdot \frac{d}{dx}(y)$$

Evaluate the derivative on the left with the chain rule.

$$(5y^4) \cdot \frac{d}{dx}(y) = 80 \cdot \frac{d}{dx}(y)$$

$$(5y^4) \cdot y' = 80 \cdot y'$$

Cancel y' and solve for y .

$$5y^4 = 80$$

$$y^4 = 16$$

$$y = \pm 2$$

Since $y = f(x)$ always lies above the x -axis, $y > 0$.

$$y = 2$$